A RELATIONSHIP BETWEEN THE STRAIN AND FAILURE OF STRESSED SOLIDS

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UDC 539.1

Since 1953 [1], S. N. Zhurkov and his collaborators have been publishing systematically the results of investigations of the temperature—time dependence of the strength of stressed solids. An enormous amount of experimental data has been gathered by now, and the appropriateness of the kinetic approach to the strength problem and the interrelationship between strain and failure has been confirmed [2]. Another relationship between strain and failure under uniaxial tensile stresses is established here. This relationship is in agreement with the described thermal fluctuation mechanism of atomic bond breaking under the action of external forces.

Plots of the logarithm of the specific strain energy as a function of the stress are linear in a fairly wide range of temperatures and stresses, where any given temperature is characterized by a specific straight-line slope. The specific strain energy is defined as the work of external forces, reduced to the volume of the solid:

$$U_s = \int_{l_0}^{l_f} \frac{F\,dl}{V}, \quad \text{or} \quad U_s = \int_{l_0}^{l_f} \sigma \frac{dl}{l},$$

where l_0 is the initial length of the unstressed specimen, and l_f is the length of the specimen after failure.

This energy, which is also referred to as the work of rupture, is used by certain authors as the failure criterion for plastic materials in a certain range of temperatures and stresses [3]. In a somewhat different form, a similar criterion, expressed in terms of the dislocations, was used earlier to determine the trends of damage and failure in heat-resistant steels [4].

At room and elevated temperatures, many structural materials fail with a smaller amount of strain as the stress is reduced, i.e., progressively smaller amounts of external energy are expended as the failure time increases, so that failure is caused largely by the internal energy of thermal atomic oscillations.

From S. N. Zhurkov's durability equation,

$$\tau = \tau_0 \exp\left[(U_0 - \gamma\sigma)/kT\right], \qquad (1)$$

where τ_0 is the oscillation period of atoms in the crystal lattice, U_0 is the failure activation energy, γ is the structure-sensitive coefficient, k is the Boltzmann constant, and T is the absolute temperature, it follows that failure occurs during the time τ_0 in the hypothetical case $\sigma = U_0/\gamma$. Failure is possible here only as a result of the work of external forces U_s, which must be equal to U_0.

Therefore, dependences of log U_s on σ for different temperatures must converge at a pole with the coordinates log U_0 and $\sigma = U_0/\gamma$.

As was done in [2] for the rate of change in creep strain, we shall assume (because of a lack of test results for structurally stable alloys and the impossibility of direct extrapolation) that the dependences of log U_s on 1/T for different stresses converge at a pole for vanishing log U_0 and 1/T = 0. Then, passing to natural logarithms, we write

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 122-126, November-December, 1976. Original article submitted December 24, 1975.

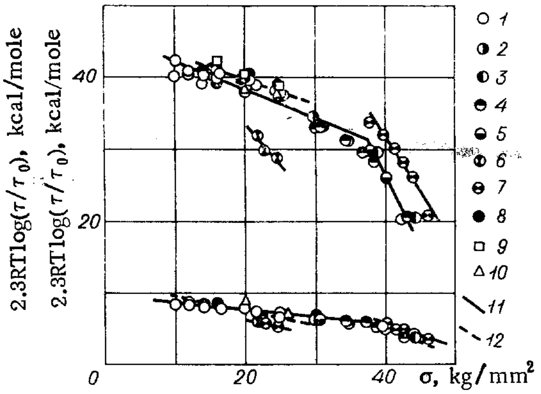

Fig. 1

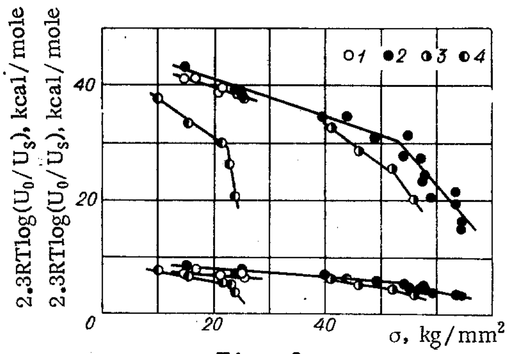

Fig. 2

Fig. 1. 1) Creep of 2.5 × 10 × 100 AK4-1T1 specimens at 130, 150, and 175°C; 2) creep of 3.5 × 10 × 100 AK4-1T1 specimens at 175°C; 3) static rupture of 3 × 10 × 100 AK4-1T1 specimens at 20 and 150°C; 4) static rupture of 2 × 10 × 100 AK4-1T1 specimens at 20, 130, 150, 175, and 200°C; 5) static rupture of 2 × 10 × 100 AK4-1T1 specimens at 25, 100, 125, 150, 175, and 200°C; 6) static rupture of 2 × 10 × 100 AK4-1 specimens (annealing at 530°C) at 130, 150, and 175°C; 7) static rupture of 3 × 10 × 100 AK4-1AT specimens at 20, 100, 125, 150, 175, and 200°C; 8) creep of diameter 10 × 100 AK4-1T1 specimens at 130, 150, and 175°C; 9) creep of diameter 8 × 60 AK4-1T1 specimens at 175°C; 10) creep of diameter 8 × 100 AK4-1T1 specimens at 150°C; 11) approximation of failure data for specimens from sheets; 12) approximation of failure data for round specimens.

Fig. 2. 1) Creep of diameter 10 × 100 D16T specimens at 150 and 175°C; 2) static rupture of diameter 8 × 80 D16T specimens at different strain rates at 20, 100, 150, 200, 250, and 300°C; 3) static rupture of 3 × 10 × 100 D16T specimens at 20, 100, 150, and 200°C; 4) static rupture of 3 × 10 × 100 D16 specimens (annealing at 500°C) at 20, 100, 150, and 200°C.

$$U_0/U_s = \exp\,[(U_0 - \gamma\sigma)/kT]\delta, \qquad (2)$$

where $\delta = \beta/\gamma$, while β is a structure-sensitive coefficient similar to γ.

If we substitute the value of the exponential from the durability equation (1) in Eq. (2), we obtain a relationship between the specific strain energy and the failure activation energy:

$$\tau^{\delta}U_s = \tau_0^{\delta}U_0 = \text{const.} \qquad (3)$$

In order to check the derived expression, we plotted 2.3RT log (τ/τ_0) and 2.3RT log (U_0/U_s) as functions of σ for the AK4-1 and D16 alloys, subjected to different types of heat treatment. (The data used were obtained in the investigations performed by A. P. Kuznetsov, V. I. Shabalin, Yu. I. Ustinovshchikov, and V. V. Evseev, and in an additional experiment performed by the author).

Data on the failure of the first and the second of these alloys are given in Figs. 1 and 2, respectively.

Since the creep tests were performed with a constant force, while the static rupture tests were performed at a constant strain rate, then, for approximate agreement with constant-stress test results, the stress values $\sigma_0 l_f/l_0$ and $\sigma_{ult} l_f/l_0$ were laid off on the axis of abscissas, while the time of exposure of the specimen to the yield stress was used as the failure time. The values U_0 = 51 kcal/mole and $\tau_0 = 10^{-13}$ sec for the alloys most closely resembling our alloys were borrowed from [2].

Comparing the curves of Figs. 1 and 2, we see the similarity between the dependences of the durability function and the specific strain energy function of the stress, i.e., the strain energy function can be obtained by multiplying the durability function by a certain coefficient δ, which is independent of the material structure. Table 1 provides the mean values of $\delta = (\log U_0 - \log U_s)/(\log \tau - \log \tau_0)$ for each material and type of heat treatment.

TABLE 1

Material	Type of test	Total no. of specimens	No. of specimens tested under same conditions	Mean value of δ
AK4-1AT	Static rupture of plate specimens	66	11	0.172
AK4-1T1	Creep of plate specimens	55	1-5	0.192
	Static rupture of plate specimens	75	3-11	0.196
	Creep of round specimens	74	1-10	0.190
	Total, specimens of all types	204		0.193
AK4-1, annealing at 530°C	Static rupture of plate specimens	29	5-6	0.189
D16T	Creep of round specimens	26	2-6	0.174
	Static rupture of round specimens at different strain rates	120	3-10	0.201
	Static rupture of plate specimens	23	5-6	0.174
	Total, specimens of all types	169		0.193
D16, annealing at 500°C	Static rupture of plate specimens	29	5-6	0.193
Total number of specimens with respect to all materials		497		0.190

In spite of the approximateness of the durability estimate for static rupture at a constant strain rate, the values of δ in these experiments do not differ substantially from the value of δ in creep. The deviations of the mean values of δ for both materials and different types of heat treatment lie within ± 10% of the average value 0.190. If we limit our considerations only to creep tests, the values of δ calculated with respect to four or more specimens "per point" also lie within the ± 10% band. The slight discrepancy between the trends of the dependences of 2.3RT log (U_0/U_s) and 2.3RT log (τ/τ_0) on σ for low stresses is explained by the fact that the processing also encompassed failure data for specimens that "lasted" 30,000 h without breaking, i.e., their durability was made too low, while the value of U_0/U_s was excessive.

The errors in determining δ do not exceed the errors in determining U_0 and γ, which have been found in [2]. Moreover, the values of δ, like U_0, can vary, depending on the degree of alloying.

Thus, the experimental results given here do not contradict relationship (3), and the dependence of the specific strain energy on the loading conditions can be described by expression (2). In the physical sense, δ probably represents the coefficient of reduction in the potential barrier of atomic bond recombination relative to the potential barrier and bond breaking. Therefore, δ may prove to be a more general constant of the material than U_0 and to be related to the type of crystal lattice of the alloy base.

The above numerical values of the coefficient δ must be rendered more accurate by using the results of constant-stress tests.

The author is grateful to A. P. Kuznetsov, V. I. Shabalin, Yu. I. Ustinovshchikov, and V. V. Evseev, who provided the test results.

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SOLUTION OF THE PLANE MIXED PROBLEM OF THE THEORY OF ELASTICITY IN THE FORM OF A SERIES IN LEGENDRE POLYNOMIALS

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UDC 539.3

By using Legendre polynomials, it is possible to demonstrate a second procedure, different from those published in the literature [1, 2], for reducing the problem to the solution of an algebraic system or to the solution of a boundary-value problem for ordinary differential equations.

1. Formulation of the Problem. The plane mixed boundary-value problem of the theory of elasticity consists in finding the functions p, q, τ, u, and v satisfying the equations

$$\partial p/\partial x + \partial\tau/\partial y + \gamma_1 = 0,\ \partial\tau/\partial x + \partial q/\partial y + \gamma_2 = 0,$$
$$p - \alpha\partial u/\partial x - \beta\partial v/\partial y = 0,\ q - \alpha\partial v/\partial y - \beta\partial u/\partial x = 0,$$
$$\tau - \mu(\partial u/\partial y + \partial v/\partial x) = 0,\ \alpha = 2\mu\,(1-\nu)/(1-2\nu),\ \nu < 1/2,\quad \mu > 0.$$
$$\beta = \alpha\nu/(1-\nu)$$

within some region Ω and taking on specified values on the boundary of the region. We shall confine ourselves to the case in which Ω is a square, $\Omega = \{x, y \mid x \in [-1, 1], y \in [-1, 1]\}$, and the boundary conditions are such that by a transformation of the desired functions the problem can be reduced to finding the functions p, q, τ, u, and v satisfying the zero boundary conditions

$$(pu)_{x=\pm1} = (qv)_{y=\pm1} = (\tau v)_{x=\pm1} = (\tau u)_{y=\pm1} = 0 \tag{1.1}$$

and the equations

$$\partial p/\partial x + \partial\tau/\partial y + f_1 = 0,\ \partial\tau/\partial x + \partial q/\partial y + f_2 = 0,$$
$$p - \alpha\partial u/\partial x - \beta\partial v/\partial y + f_3 = 0,\ q - \alpha\partial v/\partial y - \beta\partial u/\partial x + f_4 = 0,$$
$$\tau - \mu\,(\partial u/\partial y + \partial v/\partial x) + f_5 = 0,$$

where the f_σ ($\sigma = 1, \ldots, 5$) are known functions which are quadratic summable over Ω. We assume that in each of the equations (1.1) one of the multiplied functions is equal to zero all along one side of the square.

If in the case of a displacement of the square as an absolutely rigid body

$$u = a + \omega y,\ v = b - \omega x$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 126-137, November-December, 1976. Original article submitted April 14, 1976.